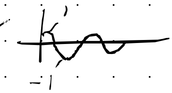
Chapter 5 Review

$y=\tan^{-1}x \to D:(-\infty,\infty)$ $R:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ ✓

$\frac{d}{dx}[\sin^{-1}u]=\frac{u'}{\sqrt{1-u^2}}$ ✓

$y=\cos^{-1}x \to D:[-1,1]$ $R:[0,\pi]$ ✓



$y=\sin^{-1}x \to D:[-1,1]$ $R:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ ✓

$A=P(1+\frac{r}{n})^{nt}$ ✓ $A=Pe^{rt}$ ✓

$\frac{d}{dx}[\log_a u] \to \frac{u'}{u(\ln(a))}$ ✓ $\int a^x\,dx \to \frac{1}{\ln a}a^x+C$ ✓

$\frac{d}{dx}[a^u] \to (\ln a)a^u \cdot u'$ ✓ $\frac{d}{dx}[a^x] \to (\ln a)a^x$ ✓

$a^x=e^{\ln a\, x}$ ✓ $b^{\log_b u}=u$ ✓ $\int a^u\,du \to \frac{1}{\ln(a)}a^u+C$ ✓

$\frac{d}{dx}[\log_a x]=\frac{1}{\ln a\, x}$ ✓ $y=\cos^{-1}x \to D:[-1,1]$ $R:[0,\pi]$ ✓

$\int \tan(u)\,du \to -\ln|\cos u|+C$ ✓ $\int a^u\,du \to \frac{1}{\ln a}a^u+C$ ✓

$\frac{d}{dx}[\log_a x] \to \frac{1}{\ln a\, x}$ $\frac{d}{dx}[\log_a u] \to \frac{u'}{u\ln a}$

→ derivative of a log with weird base is $u'/$"$u\ln a$"

$\frac{d}{dx}[\sin^{-1}u] \to \frac{u'}{\sqrt{1-u^2}}$ $y=\tan^{-1}x \to D:(-\infty,\infty)$ $R:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\int a^u\,du \to \frac{1}{\ln a}a^u+C$ ✓ $\frac{d}{dx}[\log_a x] \to \frac{1}{x\ln a}$ ✓

$\frac{d}{dx}[\log_a u] \to \frac{u'}{u(\ln(a))}$ ✓ $\frac{d}{dx}[\sin^{-1}u] \to \frac{u'}{\sqrt{1-u^2}}$ ✓

$\frac{d}{dx}[\cos^{-1}u] \to \frac{-u'}{\sqrt{1-u^2}}$ $y=\cos^{-1}x \to D:[-1,1]$ $R:[0,\pi]$ ✓

$\frac{d}{dx}[a^x] \to (\ln a)a^x$ $\int a^x\,dx \to \frac{1}{\ln a}a^x+C$ ✓

$\frac{d}{dx}[a^u] \to (\ln a)a^u u'$ ✓ $a^x = e^{\ln a x}$ ✓ $b^{\log_b u} = u$ ✓

$\int a^u du \to \frac{1}{\ln a} a^u + C$ $\frac{d}{dx}[\log_a u] \to \frac{u'}{u \ln a}$ ✗

$\frac{d}{dx}[\log_a x] \to \frac{1}{x \ln a}$ $\frac{d}{dx}[\sin^{-1} x] \to \frac{u'}{\sqrt{1-u^2}}$ ✓

$\frac{d}{dx}[a^x] \to (\ln a)a^x$ ✓ $\int \tan(u)\, du \to -\ln|\cos(u)| + C$ ✓

$\frac{d}{dx}[a^u] = (\ln a)a^u(u')$ ✗ $\frac{d}{dx}[\log_a u] \to \frac{u'}{u \ln a}$ ✓

$y = \tan^{-1} x \to D: (-\infty, \infty)$ $R: [-\frac{\pi}{2}, \frac{\pi}{2}]$ ✓ $\frac{d}{dx}[\cos^{-1} u] \to \frac{-u'}{\sqrt{1-u^2}}$ ✓

$\frac{d}{dx}[a^u] \to (\ln a)a^u(u')$ ✓ $\int e^x dx \to e^x + C$ ✓

$\frac{d}{dx}[\tan^{-1} u] \to \frac{u'}{1+u^2}$ ✓ $\int \frac{1}{x} dx \to \ln|x| + C$ ✓

$\int a^u du \to \frac{1}{\ln a} a^u + C$ ✓ $\frac{d}{dx}[\cot^{-1} u] \to \frac{-u'}{1+u^2}$ ✓

$e^a \cdot e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ ✗ $\frac{d}{dx}[\log_a x] \to \frac{1}{x \ln a}$ ✓

$\frac{d}{dx}[a^x] \to (\ln a)a^x$ ✓ $y = \cos^{-1} x \to D: [-1, 1]$ $R: [0, \pi]$ ✓

$\int a^x dx \to \frac{1}{\ln(a)} a^x + C$ ✓ $\frac{d}{dx}[\log_a u] \to \frac{u'}{u(\ln a)}$ ✓

$\frac{d}{dx}[\sec^{-1} u] \to \frac{u'}{|u|\sqrt{u^2-1}}$ ✓ $\frac{d}{dx}[a^u] \to (\ln a)a^u(u')$ ✓

$\frac{d}{dx}[\cos^{-1} u] \to \frac{-u'}{\sqrt{1-u^2}}$ ✓ $\frac{d}{dx}[\tan^{-1} u] \to \frac{u'}{1+u^2}$ ✓

$\frac{d}{dx}[\sin^{-1} u] \to \frac{u'}{\sqrt{1-u^2}}$ ✓ $\int a^u du \to \frac{1}{\ln a} a^u + C$ ✓

$\frac{d}{dx}[\cot^{-1} u] \to \frac{-u'}{1+u^2}$ ✓ $\frac{d}{dx}[a^x] \to (\ln a)a^x$ ✓

$\frac{d}{dx}[\log_a x] \to \frac{1}{x\ln a}$ ✓ $\frac{d}{dx}[\log_a u] \to \frac{u'}{u\ln a}$ ✓

$\int \tan(u)\,du \to -\ln|\cos(u)| + c$ ✓ $\frac{d}{dx}[\sec^{-1} u] \to \frac{u'}{|u|\sqrt{u^2-1}}$ ✓

$\frac{d}{dx}[a^u] \to (\ln a)a^u(u')$ ✓ $e^a \cdot e^b = e^{a+b}$ ✓ $e^a / e^b = e^{a-b}$ ✓

$\int a^x\,dx \to \frac{1}{\ln a}a^x + c$ ✓ $\int \sin(u)\,du \to -\cos(u) + c$ ✗

Integral of $\sin(u)$ is $-\cos(u) + c$ $\frac{d}{dx}[\cos^{-1}(u)] \to \frac{-u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[\tan^{-1} u] \to \frac{u'}{1+u^2}$ ✓ $\int a^u\,du \to \frac{1}{\ln a}a^u + c$ ✓

$\frac{d}{dx}[\csc^{-1} u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓ $\frac{d}{dx}[\cot^{-1} u] \to \frac{-u'}{1+u^2}$ ✓

$\frac{d}{dx}[a^u] \to \ln a\, a^u u'$ ✓ $\int \csc(u)\,du \to -\ln|\csc(u) + \cot(u)| + c$ ✗

the integral of $\csc(u)$ is negative natural log of the absolute value of $\csc u + \cot u$ $+ C$

$\frac{d}{dx}[a^x] = (\ln a)a^x$ ✓ $\frac{d}{dx}[\log_a x] \to \frac{1}{x\ln a}$ ✓ $\frac{d}{dx}[\log_a u] = \frac{u'}{u\ln a}$ ✓

$\frac{d}{dx}[\sec^{-1} u] \to \frac{u'}{|u|\sqrt{u^2-1}}$ ✓ $\int \frac{du}{\sqrt{a^2-u^2}} \to \sin^{-1}\frac{u}{a} + c$ ✓

$\int \cos u\,du \to \sin u + c$ ✓ $\frac{d}{dx}[\csc^{-1} u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓

$\frac{d}{dx}[a^u] \to (\ln a)a^u(u')$ ✓ $\int \frac{du}{a^2+u^2} \to \frac{1}{a}\tan^{-1}\frac{u}{a} + c$ ✓

$\frac{d}{dx}[\csc^{-1} u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓ $\frac{d}{dx}[\tan^{-1} u] \to \frac{u'}{1+u^2}$ ✓

$\int \sin(u)\,du \to -\cos(u) + c$ ✓ $\int \frac{du}{a^2+u^2} \to \frac{1}{a}\tan^{-1}\frac{u}{a} + c$ ✓

$\int a^u\,du \to \frac{1}{\ln a}a^u + c$ ✓ $\frac{d}{dx}[\cot^{-1} u] \to \frac{-u'}{1+u^2}$ ✓

$\int \frac{du}{\sqrt{a^2-u^2}} du \to \sin^{-1}\frac{u}{a} + c$ ✓ $e^a \cdot e^b \to e^{a+b}$ ✓ $\frac{e^a}{e^b} = e^{a-b}$ ✓

$\int \csc(u)\, du \to -\ln|\csc u + \cot u| + c$ ✗

$\int \cos u\, du \to \sin u + c$ ✓ $\frac{d}{dx}[a^u] \to \ln a\, a^u u'$ ✓

$\int \frac{du}{a^2+u^2} \to \frac{1}{a}\tan^{-1}\frac{u}{a} + c$ ✓ $\frac{d}{dx}[\csc^{-1} u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓

$\int a^u du \to \frac{1}{\ln a} a^u + c$ ✓ $\int \sin(u) du \to -\cos(u) + c$ ✓

$\frac{d}{dx}[\ln(x)] \to \frac{1}{x}$ ✓ $\int \frac{du}{u\sqrt{u^2-a^2}} \to \frac{1}{a}\sec^{-1}\frac{|u|}{a} + c$ ✓

$\int \frac{1}{u} du \to \ln|u| + c$ ✗ 1. $\cos^{-1}\frac{u}{a} + c$ ✓ 2. $\frac{1}{a}\cot^{-1}\frac{u}{a} + c$ ✓

3. $\frac{1}{a}\csc^{-1}\frac{|u|}{a} + c$ ✓ $y = e^x$ iff $x = \ln y$ ✓

$x^2 - 4x + 7 \to (-4/2)^2 \to -2^2 \to 4 \to (x^2-4x+4) + 7 - 4$

$\to (x-2)(x-2) + 3 \to (x-2)^2 + 3$ ✓

$\int \frac{du}{a^2+u^2} \to \frac{1}{a}\tan^{-1}\frac{u}{a} + c$ ✓ $\int \frac{du}{u\sqrt{u^2-a^2}} \to \frac{1}{a}\sec^{-1}\frac{|u|}{a} + c$ ✓

$\int \frac{du}{\sqrt{a^2-u^2}} \to \sin^{-1}\frac{u}{a} + c$ $\int \csc(u)\, du \to -\ln|\csc(u) + \cot(u)| + c$ ✓

$\int a^u du \to \frac{1}{\ln a} a^u + c$ ✓ 1. $\cos^{-1}\frac{u}{a} + c$ ✓ 2. $\frac{1}{a}\cot^{-1}\frac{u}{a} + c$ ✓

3. $\frac{1}{a}\csc^{-1}\frac{|u|}{a} + c$ ✗ $\frac{d}{dx}[\csc^{-1} u] \to \frac{u'}{|u|\sqrt{u^2-1}}$ ✗ $\int \frac{1}{u} du \to \ln|u| + c$ ✓

$\to (x-2)^2 + 3$ ✓ $\int \cos(u) du \to \sin(u) + c$ ✓ $g'(x) = \frac{1}{f'(g(x))}$

$\frac{d}{dx}[\csc^{-1} u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓ $\int \sin(u) du \to -\cos(u) + c$ ✓

1. $\cos^{-1}\frac{u}{a} + c$ ✓ 2. $\frac{1}{a}\cot\frac{u}{a} + c$ ✓ 3. $\frac{1}{a}\csc^{-1}\frac{|u|}{a} + c$ ✓

$\int \frac{du}{a^2+u^2} \to \frac{1}{a}\tan^{-1}\frac{u}{a}+c$ ✓ $\frac{d}{dx}[\csc^{-1}u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓

$\int \frac{du}{u\sqrt{u^2-a^2}} \to \frac{1}{a}\sec^{-1}\frac{|u|}{a}+c$ ✓ $\int \csc(u)\,du \to$

$\int \csc(u)\,du \to -\ln|\csc(u)+\cot(u)|+c$ ✓ $\to x^2-4x+7 \to (x^2-4x+\_\_)+7-\_\_$

$\to (-4/2)^2 \to -2^2 \to 4 \to (x^2-4x+4)+3 \to (x-2)(x-2)+3 \to (x-2)^2+3$ ✓

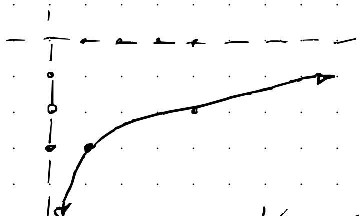
1. $\cos\frac{u}{a}+c$ ✓ 2. $\frac{1}{a}\cot^{-1}\frac{u}{a}+c$ ✓ 3. $\frac{1}{a}\csc^{-1}\frac{|u|}{a}+c$ ✓

$\frac{d}{dx}[\csc^{-1}u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓ $\int \frac{1}{u}\,du \to \ln|u|+c$ ✓

$\int \cos(u)\,du \to \sin(u)+c$ ✓

Chapter 5 Review Exercises

(1) $f(x)=\ln(x)-3$



Shifted down 3 units

ln graph

(3) $\ln\sqrt[5]{\frac{4x^2-1}{4x^2+1}} \to \ln\left(\frac{4x^2-1}{4x^2+1}\right)^{1/5} \to \frac{1}{5}\left(\ln\frac{(2x-1)(2x+1)}{4x^2+1}\right)$

$\to \frac{1}{5}\left(\left(\ln(2x-1)+\ln(2x+1)\right)-\ln 4x^2+1\right)$

(5) $\ln 3+\frac{1}{3}\ln(4-x^2)-\ln x \to \ln 3+\ln\sqrt[3]{4-x^2}-\ln x$

$\to \ln\frac{3(\sqrt[3]{4-x^2})}{x}$

(7) $g(x)=\ln\sqrt{2x} \to g(x)=\frac{1}{2}\ln 2x \to \frac{1}{2}\left(\frac{2}{2x}\right) \to \frac{1}{2x}$

(9) $f(x)=x\sqrt{\ln x} \to f'(x)=\sqrt{\ln x}+x\left(\frac{1}{2}(\ln x)^{-1/2}\left(\frac{1}{x}\right)\right)$

$\to \sqrt{\ln x}+\frac{x}{2}(\ln x)^{-1/2}\left(\frac{1}{x}\right) \to \sqrt{\ln x}+\frac{1}{2\sqrt{\ln x}}$

$\to \frac{1}{2\sqrt{\ln x}}+\left(\frac{\sqrt{\ln x}}{1}\cdot\frac{2\sqrt{\ln x}}{2\sqrt{\ln x}}\right) \to \frac{1}{2\sqrt{\ln x}}+\frac{2(\ln x)}{2\sqrt{\ln x}}$

$\to \frac{1+2(\ln x)}{2\sqrt{\ln x}}$

(11) $y = \ln\sqrt{\frac{x^2+4}{x^2-4}} \to \frac{1}{2}[(\ln x^2+4) - (\ln x^2-4)]$

$\to \frac{1}{2}\left[\left(\frac{2x}{x^2+4}\right) - \left(\frac{2x}{x^2-4}\right)\right] \to \frac{1}{2}\left[\frac{2x(x^2-4)}{x^4-16} - \frac{2x(x^2+4)}{x^4-16}\right]$

$\to \frac{1}{2}\left[\frac{2x^3-8x}{x^4-16} - \frac{2x^3+8x}{x^4-16}\right] \to \frac{1}{2}\left[\frac{-16x}{x^4-16}\right] \to \boxed{\frac{-8x}{x^4-16}}$

(13) $y = \ln(2+x) + \frac{2}{2+x}$ @ $(-1,2)$ $\to 2u^{-1}$

$\hookrightarrow u = 2+x$ $u' = 1$

$\to \frac{1}{2+x} + (-2u^{-2}) \to \frac{1}{2+x} - \frac{2}{(2+x)^2} \to \frac{2+x}{(2+x)^2} - \frac{2}{(2+x)^2}$

$\to \frac{x}{(2+x)^2} \to y'\big|_{(-1,2)} \to \frac{-1}{(2+(-1))^2} \to \frac{-1}{1^2} \to -1$

$\to y - 2 = -1(x+1) \to \boxed{y = -x+1}$

(15) $\int \frac{1}{7x-2}\,dx \to u = 7x-2$ $du = 7\,dx$

$\to \frac{1}{7}\int \frac{1}{u}\,du \to \frac{1}{7}\ln|u| + C \to \boxed{\frac{1}{7}\ln|7x-2| + C}$

$u = \cos x + 1$ $du = -\sin x\,dx$

(17) $\int \frac{\sin x}{1+\cos x}\,dx$

$\to -\int \frac{1}{u}\,du \to -\ln|u| + C \to \boxed{-\ln|\cos x + 1| + C}$

(19) $\int_1^4 \frac{2x+1}{2x}\,dx \to \int_1^4 \frac{2x}{2x}\,dx + \int_1^4 \frac{1}{2x}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx$

$\to \int_1^4 1\,dx + \int_1^4 \frac{1}{2x}\,dx \to (x) + \left(\frac{1}{2}\int \frac{1}{x}\,dx\right)$ $\frac{1}{2}(0) \to 0$

$\to \left[x + \frac{1}{2}\ln|x|\right]_1^4 \to \left(4 + \frac{1}{2}\ln 4\right) - \left(1 + \cancel{\frac{1}{2}\ln 1}\right)$

$\to 4 + \frac{1}{2}\ln 4 - 1 \to 3 + \ln\sqrt{4} \to \boxed{3 + \ln 2}$

(21) $\int_0^{\pi/3} \sec\theta\,d\theta \to \left[\ln|\sec\theta + \tan\theta|\right]_0^{\pi/3}$

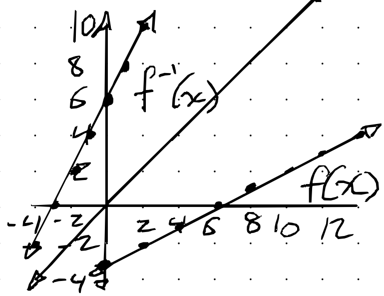
$\to \left(\ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right|\right) - \left(\ln|\sec 0 + \tan 0|\right)$

$\to \ln|2 + \sqrt{3}| - \cancel{(\ln|1+0|)} \to \ln|2+\sqrt{3}|$

(23) $f(x) = \frac{1}{2}x - 3$

(a) $x = \frac{1}{2}y - 3 \Rightarrow x + 3 = \frac{1}{2}y \Rightarrow \boxed{2x + 6 = y}$ $f^{-1}(x)$

(b)



(c) $\frac{1}{2}(2x+6) - 3 \Rightarrow x + 3 - 3 \Rightarrow x$

$2(\frac{1}{2}x - 3) + 6 \Rightarrow x - 6 + 6 \Rightarrow x$

(d) f $D: (-\infty, \infty)$ $R: (-\infty, \infty)$ f^{-1} $D: (-\infty, \infty)$ $R: (-\infty, \infty)$

(27) $f(x) = \sqrt[3]{x+1}$

$\Rightarrow x = \sqrt[3]{y+1} \Rightarrow x^3 = y + 1 \Rightarrow y = x^3 - 1$

(29) $f(x) = x^3 + 2$, $a = -1$

$f(?) = -1$ $f^{-1}(-1) = ?$

$-1 = x^3 + 2 \Rightarrow -3 = x^3 \Rightarrow \sqrt[3]{-3} = x$

$\Rightarrow f((-3)^{1/3}) = -1$ $f^{-1}(-1) = (-3)^{1/3}$

$\Rightarrow 3x^2$ $\frac{1}{3(-3^{1/3})^2} \Rightarrow \frac{1}{3(-3^{2/3})}$ $f'(-3^{1/3})$

$\rightarrow 3(-3^{1/3})^2 \Rightarrow 3(-3^{2/3})$

Formula Check

$\int \sin(u)\,du = -\cos u + C$ ✓ $\frac{d}{dx}[\sin^{-1} u] \Rightarrow \frac{u'}{\sqrt{1-u^2}}$ ✓

$\frac{d}{dx}[\csc^{-1} u] \Rightarrow \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓ $g'(x) = \frac{1}{f'(g(x))}$ ✓

$A = P\cdot(1+\frac{r}{n})^{nt}$ $A = Pe^{rt}$ ✓ $\frac{d}{dx}[\cot^{-1} u] \to \frac{-u'}{1+u^2}$ ✓

$\frac{d}{dx}[\cos^{-1} u] \to \frac{-u'}{\sqrt{1-u^2}}$ ✓ $e^a \cdot e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ ✓

$\frac{d}{dx}[\log_a u] = \frac{u'}{u(\ln a)}$ ✓ $\frac{d}{dx}[\sec^{-1} u] \to \frac{u'}{|u|\sqrt{u^2-1}}$ ✓

$\int e^u du \to e^u \cdot u' + C$ ✓ $\int \sec(u)\, du = \ln|\sec u + \tan u| + C$ ✓

$y = \cos^{-1} u$ $D: [-1, 1]$ $R: [0, \pi]$ ✓ $y = \sin^{-1} x$ $D: [-1, 1]$ $R: [-\frac{\pi}{2}, \frac{\pi}{2}]$ ✓

$\frac{d}{dx}[\tan^{-1} u] \to \frac{u'}{1+u^2}$ ✓ 1: $\cos^{-1} \frac{u}{a} + C$ ✓ 2. $\frac{1}{a}\cot^{-1}\frac{u}{a} + C$ ✓

3. $\frac{1}{a}\csc^{-1}\frac{|u|}{a} + C$ ✓ $\int \tan(u)\, du \to -\ln|\cos(u)| + C$ ✓

$x^2 - 4x + 7 \to (x^2 - 4x + \_\_) + 7 - \_\_ \to (-4/2)^2 \to -2^2 \to 4$

$\to (x^2 - 4x + 4) + 7 - 4 \to (x-2)^2 + 3$ ✓ $\int \frac{1}{x} dx \to \ln|x| + C$ ✓

$\frac{d}{dx}[\log_a x] \to \frac{1}{x \ln a}$ ✓ $\int \frac{du}{u\sqrt{u^2 - a^2}} \to \frac{1}{a}\sec^{-1}\frac{|u|}{a} + C$ ✓

$\frac{d}{dx}[a^u] \to (\ln a) a^u (u')$ ✓ $\int e^x dx \to e^x + C$ ✓

$\frac{d}{dx}[\sin^{-1} u] \to \frac{u'}{\sqrt{1-u^2}}$ ✓ $\log_b a \to \frac{\ln a}{\ln b}$

One to one (passes both horizontal & vertical line tests) ✓

Monotonic (strictly increasing or decreasing) ✓

$\int \frac{1}{u} du \to \ln|u| + C$ ✓ $\int \cos(u)\, du \to \sin u + C$ ✓

$\frac{d}{dx}[\csc^{-1} u] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓ $\frac{d}{dx}[\cos^{-1} u] \to \frac{-u'}{\sqrt{1-u^2}}$ ✓

$e^a \cdot e^b = e^{a+b}$ ✓ $\frac{e^a}{e^b} = e^{a-b}$ ✓ $\frac{d}{dx}[\cot^{-1} u] \to \frac{-u'}{1+u^2}$ ✓

$\frac{1}{f'(g(x))}$ ✓ $A=P(1+\frac{r}{n})^{nt}$ ✓ $A=Pe^{rt}$ ✓

$\frac{d}{dx}[\log_a u] \to \frac{u'}{u(\ln(a))}$ ✓ $y=\cos^{-1}x$ $D:[-1,1]$ $R:[0,\pi]$ ✓

$\int \csc(u)\,du \to -\ln|\csc u+\cot u|+c$ ✓ $\frac{d}{dx}[\ln u] \to \frac{u'}{u}$ ✓

$\int \sec(u)\,du \to \ln|\sec(u)+\tan(u)|+c$ ✓ $y=\sin^{-1}x$ ✓ $D:[-1,1]$ $R:[-\frac{\pi}{2},\frac{\pi}{2}]$

$\frac{d}{dx}[\tan^{-1}u] \to \frac{u'}{1+u^2}$ ✓ $\frac{d}{dx}[e^x] \to e^x$ ✓ $\frac{d}{dx}[e^u] \to e^u \cdot u'$ ✓

$a^x=e^{(\ln a)x}$ ✓ $b^{\log_b u}=u$ ✓ $\int \tan(u)\,du \to -\ln|\cos(u)|+c$ ✓

1. $\cos^{-1}\frac{u}{a}+c$ ✓ 2. $\frac{1}{a}\cot^{-1}\frac{u}{a}+c$ ✓ 3. $\frac{1}{a}\csc^{-1}\frac{|u|}{a}+c$ ✓

$\int \frac{du}{\sqrt{a^2-u^2}} \to \sin^{-1}\frac{u}{a}+c$ ✓ $\int \frac{du}{u\sqrt{u^2-a^2}} \to \frac{1}{a}\sec^{-1}\frac{|u|}{a}+c$ ✓

$\frac{d}{dx}[\ln x] \to \frac{1}{x}$ ✓ $\int a^u\,du \to \frac{1}{(\ln(a))}a^u+c$ ✓

$\frac{d}{dx}[a^x] \to (\ln a)a^x$ ✓ $\int a^x\,dx \to \frac{1}{\ln a}a^x+c$ ✓

$\int \cot u\,du \to \ln|\sin(u)|+c$ ✓ $y=\tan^{-1}x$ $D:(-\infty,\infty)$ ✓ $R:[-\frac{\pi}{2},\frac{\pi}{2}]$ ✓

$\frac{d}{dx}(y=x^{x-1}) \to \ln y = x-1\ln x$

$\to \frac{y'}{y}=((x-1)(\frac{1}{x}))+((1)(\ln x))$

$\to y'=y(x-1(\frac{1}{x})+\ln x) \to y'=x^{x-1}(x-1(\frac{1}{x})+\ln x)$ ✓

$y=e^x$ <u>iff</u> $x=\ln y$ ✓ $\int \frac{du}{a^2+u^2}=\frac{1}{a}\tan^{-1}\frac{u}{a}+c$ ✓

$\int \sin(u)\,du \to -\cos u+c$ ✓ $\frac{d}{dx}[\sin^{-1}u] \to \frac{u'}{\sqrt{1-u^2}}$ ✓

$\int \csc(u)\,du=-\ln|\csc u+\cot u|+c$ ✓ $\int \sec(u)\,du=\ln|\sec u+\tan u|+c$ ✓

$\frac{d}{dx}[\ln(u)] \to \frac{u'}{u}$ ✓ $\frac{d}{dx}[\cos^{-1}u] \to \frac{-u'}{\sqrt{1-u^2}}$ ✓

$\int \tan(u)\,du \rightarrow -\ln|\cos(u)|+c$ ✓

$\frac{d}{dx}[\csc^{-1} u] \rightarrow \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓

$\int \cot(u)\,du \rightarrow \ln|\sin(u)|+c$ ✓

$\int \csc(u)\,du \rightarrow -\ln|\csc u+\cot u|+c$ ✓

$\int \sin(u)\,du \rightarrow -\cos u+c$ ✓

$\int \sec(u)\,du \rightarrow \ln|\sec(u)+\tan(u)|+c$ ✓

Chapter 5 Quiz Review

ln differentiation: $y=x^{(x-1)}$

↳ you have a function of x to the power of a function of x: $y=g(x)^{f(x)}$

1. take ln of both sides
2. differentiate both sides
3. isolate y' → 4. replace y with given

(10) $\int \frac{x^2-3x+9}{x-3}\,dx$ If numerator's degree is ≥ degree of denom, use division

$\rightarrow$ 3 | 1   −3   9 ; 3   0 ; 1   0 | 9 $\rightarrow \int x\,dx + 9\int \frac{1}{x-3}\,dx$ synth. division

$\rightarrow \boxed{\frac{x^2}{2}+9\ln|x-3|+C}$

(12) $\int 8^{3x}\,dx$ $u=3x$ $du=3\,dx$

$\rightarrow \frac{1}{3}\int a^u\,du \rightarrow \frac{1}{3}\left(\frac{1}{\ln 8}\right)8^{3x}+C \rightarrow \frac{1}{3(\ln 8)}8^{3x}+C$

⑧ $\int x\csc(x^2)\,dx \qquad u=x^2 \qquad du=2x\,dx$

$\frac{1}{2}\int \csc(u)\,du \to -\frac{1}{2}\ln|\csc u+\cot u|+c$

$\to -\frac{1}{2}\ln|\csc x^2+\cot x^2|+C$ replace the "u" from u substitution

⑨ $\int \frac{-4}{x^2+6x+13}\,dx \to (x^2+6x+9)+13-9$

$\to (x+3)(x+3)+4$

$\to \int \frac{-4}{(x+3)^2+4}\,dx$

$\frac{1}{a}\tan^{-1}\frac{u}{a}+C$

$\to u=x+3 \qquad a=2 \qquad u'=1 \qquad du=1\,dx$

$-4\left(\frac{1}{2}\tan^{-1}\frac{x+3}{2}\right)+C \to -2\tan^{-1}\left(\frac{x+3}{2}\right)+C$

⑯ $\int -3\sec x\,dx \qquad u=x \quad du=1\,dx$

$\to -3\int \sec x\,dx \qquad -3\ln|\sec x+\tan x|+C$

AP

$f(x)=2xe^{2x}$

ⓐ find $\lim_{x\to\infty} f(x)$ & $\lim_{x\to-\infty} f(x)$

$\to \lim_{x\to-\infty}=0 \qquad \lim_{x\to\infty}=\infty$

ⓑ $f'(x)=2e^{2x}+2x(2)(e^{2x})=2e^{2x}(1+2x)$

$f'(-\frac{1}{2})=0 \qquad f(-\frac{1}{2})=-\frac{1}{e}$

$f'(x)<0$ on $x<-\frac{1}{2}$, $f'(x)>0$ on $x>-\frac{1}{2}$

ⓒ $[-\frac{1}{e},\infty)$

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1+bx)$

$y'(-\frac{1}{b}) = 0$ @ $x = -\frac{1}{b}$ $y = -\frac{1}{e}$

$y = bxe^{bx}$ y has abs. min. of $-\frac{1}{e}$ for all nonzero "b"

Formula review

$\frac{d}{dx}(\ln(x)) \Rightarrow \frac{1}{x}$ $\frac{d}{dx}(\ln(u)) \Rightarrow \frac{u'}{u}$

$\int \frac{1}{x}dx \Rightarrow \ln|x| + c$ ✓ $\int \frac{1}{u}du \Rightarrow \ln|u| + c$ ✓

$\frac{d}{dx}(y = x^{x-1}) \Rightarrow \ln y = (x-1)\ln x$

$\Rightarrow \frac{y'}{y} = (x-1)(\frac{1}{x}) + (1)(\ln x)$

$\Rightarrow \frac{y'}{y} = \frac{x-1}{x} + \ln x \Rightarrow y' = y(\frac{x-1}{x} + \ln x)$

$y' = (x^{x-1})(\frac{x-1}{x} + \ln x)$

$\int \sin(u)du \Rightarrow -\cos u + c$ ✓ $\int \cos u du \Rightarrow \sin u + c$ ✓

$\int \tan u du = -\ln|\cos u| + c$ ✓ $\int \cot u du \Rightarrow \ln|\sin u| + c$ ✓

$\int \sec u du \Rightarrow \ln|\sec u + \tan u| + c$ ✓ One-to-one ✓

Monotonic ✓

$\int \csc u du \Rightarrow -\ln|\csc u + \cot u| + c$ ✓

$g'(x) = \frac{1}{f'(g(x))}$ ✓ $y = e^x$ iff $x = \ln y$ ✓

$e^a \cdot e^b = e^{a+b}$ ✓ $\cancel{\frac{e^a}{e^b}} = e^{a-b}$ ✓ $\frac{d}{dx}e^x \Rightarrow e^x$ ✓

$\frac{d}{dx}e^u = e^u u'$ ✓ $\int e^x dx \Rightarrow e^x + c$ $\int e^u du \Rightarrow e^u \cdot u' + c$ ✓

$\int \frac{1}{x}dx \Rightarrow \ln|x| + c$ ✓ $\frac{d}{dx}(\ln(u)) \Rightarrow \frac{u'}{u}$ ✓

$a^x = e^{\ln a x}$ ✓ $b^{\log_b u} = u$ ✓

$\int \frac{1}{u} du \Rightarrow \ln|u| + C$ $\int e^x dx \Rightarrow e^x + C$ ✓

$\int e^u du \Rightarrow e^u u' + C$ ✓ $\int \cos u \, du \Rightarrow \sin u + C$

$\int \tan u \, du \Rightarrow -\ln|\cos u| + C$ ✓ $\int \sin(u) du \Rightarrow -\cos u + C$ ✓

$\int \cot(u) du \Rightarrow \ln|\sin(u)| + C$ ✓ $\int \csc u \, du \Rightarrow -\ln|\csc u + \cot u| + C$ ✓

$\int \sec u \, du \Rightarrow \ln|\sec(u) + \tan(u)| + C$ ✓ $g'(x) = \frac{1}{f'(g(x))}$ ✓

monotonic ✓ one-to-one ✓ $y = e^x$ ~~iff~~ $x = \ln|y|$

$\frac{d}{dx} e^x \Rightarrow e^x$ ✓ $e^a \cdot e^b = e^{a+b}$ ✓ $\frac{e^a}{e^b} = e^{a-b}$ ✓

$\frac{d}{dx} e^u \Rightarrow e^u u'$ ✓ $\frac{d}{dx}(\ln x) \Rightarrow \frac{1}{x}$ ✓ $\int \frac{1}{u} du \Rightarrow \ln|u| + C$ ✓

$\frac{d}{dx}(y = x^{x-1}) \Rightarrow$ $y' = x^{x-1}\left(\frac{x-1}{x} + \ln x\right)$

$\int \frac{1}{x} dx \Rightarrow \ln|x| + C$ ✓ $\frac{d}{dx}(\ln(u)) \Rightarrow \frac{u'}{u}$ ✓

$a^x = e^{\ln a x}$ ✓ $b^{\log_b u} = u$ ✓ $\int e^x dx \Rightarrow e^x + C$ ✓

$\int e^u du \Rightarrow e^u u' + C$ ✓ $\int \cos(u) du \Rightarrow \sin(u) + C$ ✓

$\int \sin(u) du \Rightarrow -\cos u + C$ ✓ $\int \tan(u) du \Rightarrow -\ln|\cos(u)| + C$ ✓

$\int \csc(u) du \Rightarrow -\ln|\csc u + \cot| + C$ ✓ $\int \cot u \, du \Rightarrow \ln|\sin(u)| + C$ ✓

$\log_b a \Rightarrow \frac{\log_{\text{desired base}} a}{\log_{\text{desired base}} b}$ ✓ $\frac{d}{dx}[a^x] \Rightarrow (\ln a) a^x$

$\frac{d}{dx}[a^u] \Rightarrow (\ln a) a^u (u')$ $\frac{d}{dx}[\log_a x] \Rightarrow \frac{1}{\ln a \, x}$ ✓

$\frac{d}{dx}[\log_a u] \Rightarrow \frac{u'}{u(\ln a)}$ ✓ $\int a^x dx \Rightarrow \frac{1}{\ln a} a^x + C$ ✓

$\frac{d}{dx}[a^x] \to (\ln a)a^x$ ✓ $\frac{d}{dx}[a^u] \to (\ln a)a^u(u')$ ✓

$\int a^u\,du \to \frac{1}{\ln a}a^u + C$ ✓ $A = Pe^{rt}$ ✓

$A = P(1+\frac{r}{n})^{nt}$ ✓

$y = \sin^{-1}(x) \to$ D: $[-1,1]$ ✓ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ ✓

$y = \cos^{-1}x$ D: $[-1,1]$ ✓ R: $[0,\pi]$ ✓

$y = \tan^{-1}x$ D: $(-\infty,\infty)$ R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ ✓

$\frac{d}{dx}[\sin^{-1}(u)] \to \frac{u'}{\sqrt{1-u^2}}$ ✓ $\frac{d}{dx}[\cos^{-1}u] \to \frac{-u'}{\sqrt{1-u^2}}$ ✓

$\frac{d}{dx}[\tan^{-1}x]$ $\frac{u'}{1+u^2}$ ✓ $\frac{d}{dx}[\cot^{-1}u] \to \frac{-u'}{1+u^2}$ ✓

$\log_b a \to \frac{\log_{db} a}{\log_{db} b}$ ✓ $\frac{d}{dx}[\sec^{-1}u] \to \frac{u'}{|u|\sqrt{u^2-1}}$ ✓

$\frac{d}{dx}[\log_a x] \to \frac{1}{\ln a\, x}$ ✓ $\frac{d}{dx}[\log_a u] \to \frac{u'}{u \ln a}$ ✓

$\boxed{\frac{d}{dx}[a^u] \to (\ln a)a^u(u')}$ $\int a^x dx \to \frac{1}{\ln a}a^x + C$ ✓

$\frac{d}{dx}[a^x] \to \ln a\; a^x$ ✓ $\frac{d}{dx}[\csc^{-1}x] \to \frac{-u'}{|u|\sqrt{u^2-1}}$ ✓

$A = Pe^{rt}$ ✓ $A = P(1+\frac{r}{n})^{nt}$ ✓ $\int a^u\,du \to \frac{1}{\ln a}a^u + C$ ✓

$\frac{d}{dx}[a^u] \to (\ln a)a^u(u')$ ✓ $\int \frac{du}{\sqrt{a^2-u^2}} \to \sin^{-1}\frac{u}{a} + C$

$\int \frac{du}{a^2+u^2} \to \frac{1}{a}\tan^{-1}\frac{u}{a} + C$ ✓

$\int \frac{du}{u\sqrt{u^2-a^2}} \to \frac{1}{a}\sec^{-1}\frac{|u|}{a} + C$ ✓

1. $\cos^{-1}\frac{u}{a} + C$ ✓

$\frac{d}{dx}[\sin^{-1}u] \to \frac{u'}{\sqrt{1-u^2}}$ ✓

$-4/2 \to (-2)^2 \to 4$

2. $\frac{1}{a}\cot^{-1}\frac{u}{a} + C$ ✓

3. $\frac{1}{a}\csc^{-1}\frac{|u|}{a} + C$ ✓

$x^2 - 4x + 7$

$\to (x^2 - 4x + \underline{4}) + 7 - 4 \to (x-2)^2 + 3$

$\frac{d}{dx}\ln x \to \frac{1}{x}$ $\frac{d}{dx}\ln u \to \frac{u'}{u}$

$\int \frac{1}{x}\,dx \to \ln(x)+C$ $\int \frac{1}{u}\,du \to \ln|u|+C$

$\to y' = x^{x-1}\left(\frac{x-1}{x}+\ln x\right)$ $\int \sin(u)\,dx \to -\cos(u)+C$

$\int \cos u\,du \to \sin u + C$ $\int \tan(u)\,du \to -\ln|\cos(u)|+C$

$\int \cot u\,du \to \ln|\sin u|+C$ $\int \sec u\,du \to \ln|\sec u+\tan u|+C$

$\int \csc(u)\,du \to -\ln|\csc u+\cot u|+C$ monotonic, one-to-one

$g'(x) = \frac{1}{f'(g(x))}$ $y=e^x$ iff $x=\ln(y)$

$e^a \cdot e^b = e^{a+b}$ $e^a/e^b = e^{a-b}$ $\frac{d}{dx}(e^x) \to e^x$

$\frac{d}{dx}e^u \to e^u u'$ e^x+C $e^u u'+C$

$a^x = e^{\ln a\, x}$ $b^{\log_b u} = u$ $\frac{\log_d a}{\log_d b}$

$\ln a\; a^x$ $(\ln a)a^u(u')$

$\frac{1}{\ln a}$ $\frac{u'}{u\ln a}$ $\int a^x\,dx \to \frac{1}{\ln a}a^x+C$

$\frac{1}{\ln a}a^u+C$ $A=Pe^{rt}$ $A=P\left(1+\frac{r}{n}\right)^{nt}$

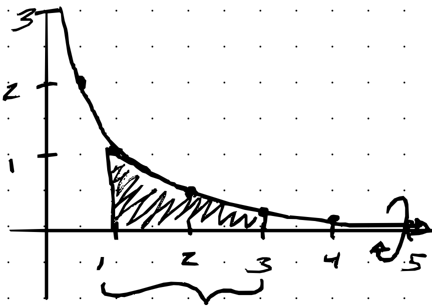
$\frac{-u'}{\sqrt{1-u^2}}$ $\frac{-u'}{1+u^2}$ $\frac{-u'}{|u|\sqrt{u^2-1}}$

$\sin^{-1}\frac{u}{a}+C$ $\frac{1}{a}\tan^{-1}\frac{u}{a}+C$ $\frac{1}{a}\sec^{-1}\frac{|u|}{a}+C$

$\frac{d}{dx}[\log_a u] \to \frac{1}{\ln a\, x}$

Ch 7 Practice

$y=\frac{1}{x}$, x-axis, $x=1,3$



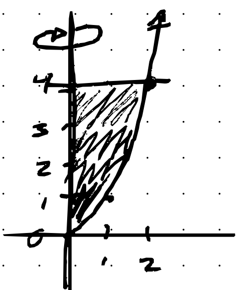
$$V=\pi\int_1^3 \left(\frac{1}{x}\right)^2 dx$$

$$\to V=\pi\int_1^3 x^{-2}\,dx$$

$$\to V=\pi\,\frac{x^{-1}}{-1}\Big|_1^3 \to \pi\,\frac{-1}{x}\Big|_1^3$$

$$\to V=\pi\left[-\frac{1}{3}+\frac{1}{1}\right] \to \pi\,\frac{2}{3} \text{ or } \boxed{\frac{2\pi}{3}}$$

$y=x^2$, $x=0$, $y=4$, y-axis



$$x=\pm\sqrt{y}$$

$$V=\pi\int_0^4 R^2(y)\,dy \to V=\pi\int_0^4 (\sqrt{y})^2\,dy$$

$$\to \pi\int_0^4 y\,dy \to \pi\,\frac{y^2}{2}\Big|_0^4$$

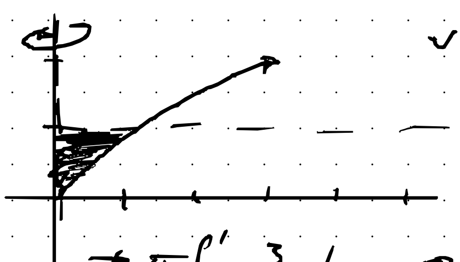
$$\to \pi\left[\frac{16}{2}-\frac{0}{2}\right] \to \boxed{8\pi}$$

$$y=x^{2/3}$$

$$y^3=x^2$$

$$x=\pm\sqrt{y^3}$$

$y=x^{2/3}$, $x=0$, $y=1$, y-axis



$$V=\pi\int_0^1 R^2(y)\,dy \to$$

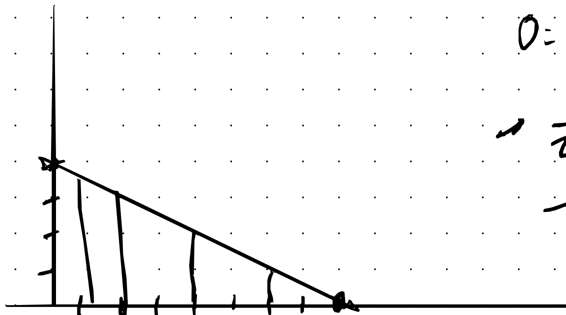
$$V=\pi\int_0^1 \sqrt{y^3}^{\,2}\,dy$$

$$\to \pi\int_0^1 y^3\,dy \to \pi\,\frac{y^4}{4}\Big|_0^1 \to \pi\left[\frac{1}{4}-\frac{0}{4}\right]$$

$$\to \boxed{\frac{\pi}{4}}$$

$x=0,\ y=0,\ y=4-\frac{1}{2}x$ use semicircles perpendicular to the x-axis.

$A_{\ominus}=\frac{1}{2}\pi r^2$



$0=4-\frac{1}{2}x$

$\frac{1}{2}x=4$

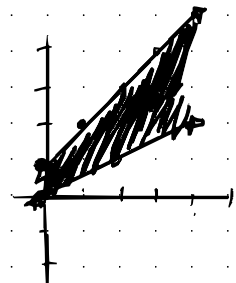
$x=8$

$V=\frac{\pi}{2}\int_0^8 \left(2-\frac{1}{4}x\right)^2 dx$

≈ 16.755

$\left(\frac{1}{2}\left(4-\frac{1}{2}x\right)\right)^2$

(7) $\int_0^4 \left[(x+1)-\frac{x}{2}\right] dx \to$



(3) $f(x)=x(x^2-3x+3)\quad g(x)=x^2$

$x^3-3x^2+3x=x^2 \to 0=x^3-4x^2+3x$

$x=0,1$

$A=\int_0^1 \left((x^3-3x^2+3x)-(x^2)\right) dx$

$\int_0^1 (x^3-3x^2+3x-x^2)\, dx \to \int_0^1 (x^3-4x^2+3x)\, dx$

$\to \left[\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}\right]_0^1 \to \left[\left(\frac{1}{4}-\frac{4}{3}+\frac{3}{2}\right)-(0-0+0)\right]$

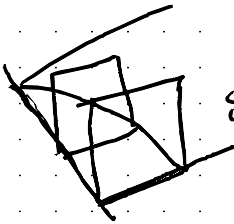
$\to \left(\frac{3}{12}-\frac{16}{12}+\frac{18}{12}\right) \to \frac{3}{12}+\frac{2}{12}=\frac{5}{12}$

$\int_1^3 \left(x^2-(x^3-3x^2+3x)\right) dx \to \int_1^3 (-x^3+4x^2-3x)\, dx$

$\left[-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\right]_1^3 \to \left[\left(\frac{81}{4}+\frac{108}{3}-\frac{27}{2}\right)-\left(\frac{-1}{4}+\frac{4}{3}-\frac{3}{2}\right)\right]$

$\frac{81}{4}+\frac{108}{3}-\frac{27}{2}+\frac{1}{4}-\frac{4}{3}+\frac{3}{2}+\frac{5}{12} \to \frac{811}{12}$

$y=\sqrt{x}$, $x=4$, $y=0$, squares with base perpendicular to x-axis.

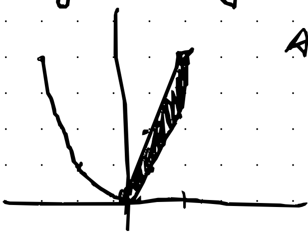


$$V=\int_0^4 \sqrt{x}^{\,2}\,dx$$

$$SqB=\sqrt{x}$$

$$V=\int_0^4 x\,dx \to \frac{x^2}{2}\Big|_0^4 \to \left[\frac{16}{2}-\frac{0}{2}\right] \to \boxed{8}$$

$y=x^2$ $y=2x$ $(0,2)$

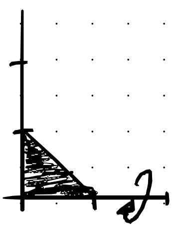


$$A=\int_0^2 (2x-x^2)\,dx$$

$$\left[x^2-\frac{x^3}{3}\right]_0^2 \to \left[\left(4-\frac{8}{3}\right)-\cancel{(0-0)}\right]$$

$$4-\frac{8}{3} \to \frac{12}{3}-\frac{8}{3} \to \boxed{\frac{4}{3}} \text{ or } 1.\overline{33}$$

(1) $y=-x+1$



$$V=\pi\int_0^1 \left((-x+1)^2-0^2\right)dx$$

$(-x+1)$

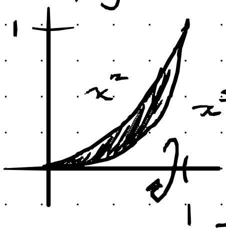
$$\to \pi\int_0^1 (x^2-2x+1)\,dx$$

$$\pi\left[\frac{x^3}{3}-x^2+x\right]_0^1 \to \pi\left[\left(\frac{1}{3}\right)-(0)\right]$$

$$\to \frac{\pi}{3}$$

$(0,1)$

(5) $y=x^2$, $y=x^5$



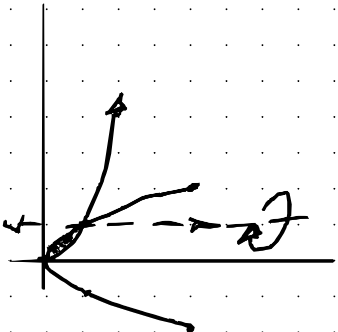
$$V=\pi\int_0^1 \left((x^2)^2\right)dx-\pi\int_0^1 \left((x^5)^2\right)dx$$

$$\to \pi\left[\frac{x^5}{5}\right]_0^1-\pi\left[\frac{x^{11}}{11}\right]_0^1$$

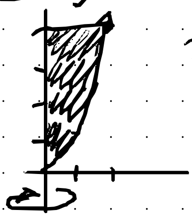
$$\to \frac{\pi}{5}-\frac{\pi}{11} \to \frac{22\pi}{110}-\frac{10\pi}{110} \to \frac{12\pi}{110}$$

$$\frac{6\pi}{55}$$

$y = x^2$, $x = y^2$, about $y = 1$ $\boxed{dx}$



(7) $y = x^2$ $\pm\sqrt{y} = x$ $(0, 4]$

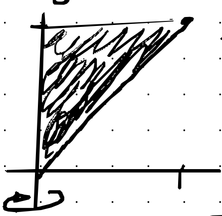


$$V = \pi \int_0^4 \left((\sqrt{y})^2 - 0^2 \right) dy \qquad \pi \int_0^4 y \, dy$$

$$\to \pi \left[\frac{y^2}{2} \right]_0^4 \qquad \pi \left[\frac{16}{2} - 0 \right] \to \boxed{8\pi}$$

(9) $y = x^{2/3} \to \sqrt[3]{x^2} \to y^3 = x^2 \to x = \sqrt{y^3}$ or $y^{3/2}$

$(0, 1]$

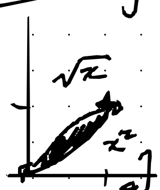


$$V = \pi \int_0^1 \left((\sqrt{y^3})^2 - 0^2 \right) dy$$

$$\to \pi \int_0^1 y^3 \, dy \to \pi \left[\frac{y^4}{4} \right]_0^1 \to \pi \left[\left(\frac{1}{4} \right) - 0 \right]$$

$$\to \boxed{\frac{\pi}{4}}$$

Ex3: $y = x^2$ $y = \sqrt{x}$



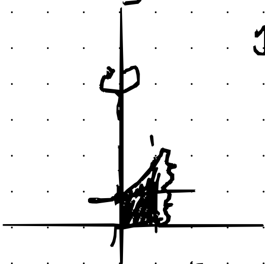
$$V = \pi \int_0^1 \left((\sqrt{x})^2 - (x^2)^2 \right) dx$$

$$\to \pi \int_0^1 (x - x^4) \, dx \to \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$$

$$\to \pi \left[\left(\frac{1}{2} - \frac{1}{5} \right) - 0 \right] \to \pi \left[\frac{5}{10} - \frac{2}{10} \right] \to \frac{3\pi}{10}$$

Ex 4: $y = x^2+1,\ y=0,\ x=0,\ x=1$

$y-1 = x^2 \to \sqrt{y-1} = x$



$$V = \pi\int_1^2 \left(1^2 - (\sqrt{y-1})^2\right) dy + \pi\int_0^1 (1^2)\, dy$$

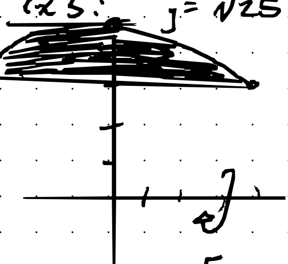
$$\to \pi\int_1^2 \left(1-(y-1)\right) dy + \pi\int_0^1 1\, dy$$

$$\to \pi\left[y - \frac{y^2}{2} + y\right]_1^2 + \pi\left[y\right]_0^1 \to \pi\left[(4-2)-\left(2-\frac{1}{2}\right)\right] + \pi[1]$$

$$2y - \frac{y^2}{2} \qquad \to \pi\left[2 - \frac{3}{2}\right] + \pi$$

$$\to \frac{\pi}{2} + \pi \to \boxed{\frac{3\pi}{2}}$$

Ex 5: $y = \sqrt{25-x^2}$ $\quad y = 3 \quad$ @ x-axis



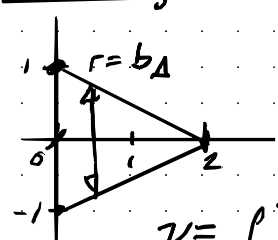
$$V = \pi\int_{-4}^{4} \left(\sqrt{25-x^2}^{\,2} - 3^2\right) dx$$

$$\to \pi\int_{-4}^{4} \left(25-x^2-9\right) dx \to \pi\int_{-4}^{4} \left(16-x^2\right) dx$$

$$\to \pi\left[16x - \frac{x^3}{3}\right]_{-4}^{4} \quad \pi\left[\left(64-\frac{64}{3}\right)+\left(+64+\left(\frac{-64}{3}\right)\right)\right]$$

$$\to \pi\left[\left(\frac{128}{3}\right) + \frac{128}{3}\right] \to \frac{256\pi}{3}$$

Ex 6: $f(x) = 1 - \frac{x}{2}$ $\quad g(x) = -1 + \frac{x}{2}$ $\quad x = 0$ $\qquad (0,2)$



$$A_\Delta = \frac{\sqrt{3}}{4} b^2$$

$$b = 1 - \frac{x}{2} - \left(-1 + \frac{x}{2}\right)$$

$$\to 2 - x \qquad A(x) = \frac{\sqrt{3}}{4}(2-x)^2$$

$$V = \int_0^2 \left(\frac{\sqrt{3}}{4}(2-x)^2\right) dx$$

Practice for Mock Exam

① $(2x^2+5)^7$ $\quad u = 2x^2+5 \to u^7$

$\to 7(2x^2+5)^6(4x) \to 28x(2x^2+5)^6$

② $\int \frac{1}{3x+12}\,dx \quad \frac{1}{3}\int \frac{1}{u}\,du$ $\quad u = 3$

$\frac{1}{3}\ln \frac{1}{3x+12}$

③ $\frac{5-x}{x^3+2} \to \frac{-1}{3x^2}$ $\quad \frac{(x^3+2)(-1)-(5-x)(3x^2)}{(x^3+2)^2}$

$-x^3-$

$+3x^3 \to 2x^3$

④ $A_{\text{trapezoid}} = \frac{1}{2}(h)(b_1+b_2)$

$A = \frac{1}{2}\left(\frac{1}{2}\right)(80) + \frac{1}{2}\left(\frac{3}{2}\right)(60+40) + \frac{1}{2}(1)(40+30)$

$\frac{1}{4}80 + \frac{3}{4}(100) + \frac{1}{2}70$

$20 + 75 + 35 \to 130$

⑤ $\sin(x^2+\pi)$ $\quad f'(\sqrt{2\pi})$

$\to u = x^2+\pi \to \cos(x^2+\pi)(2x)$

$\to 2x$

$\to \cos(2\pi+\pi)(2\sqrt{2\pi})$

$\to \cos(3\pi)(2\sqrt{2\pi})$

$-1(2\sqrt{2\pi})$

(6) $3x^2 - x^3 \rightarrow \cancel{f' = 6x - 3x^2}$

$\cancel{f'(1) = 6 - 3 \rightarrow 3}$ $\rightarrow$ $-42/2$

$\cancel{f'(5) = 30 - 75 \rightarrow 45}$ $\rightarrow -21$

$f(1) = 3 - 1 \rightarrow 2$

$f(5) = 75 - 125 \rightarrow -50 \rightarrow \frac{-50-2}{5-1} \rightarrow \frac{-52}{4}$

$\rightarrow -26 \rightarrow -13$

(8) $f(x) = e^{x/3} \rightarrow f'(x) = e^{x/3}\left(\frac{1}{3}\right)$

$\frac{e^{x/3}}{3}$ of $(3\ln 4) \rightarrow \frac{e^{\frac{\cancel{3}\ln 4}{\cancel{3}}}}{3} \rightarrow \frac{4}{3}$

$y - 4 = \frac{4}{3}(x - 3\ln 4)$

(10) $\int_0^2 (x^3+1)^{1/2} x^2 \, dx \rightarrow \int_0^2 \sqrt{x^3+1} \; x^2 \, dx$

(11) $x^2 + xy - 3y = 3 \rightarrow x^2 + xy - 3y - 3 = 0$

$2x + \left(x\frac{dy}{dx} + 1(y)\right) - 3\frac{dy}{dx} = 0$ $\quad 1 \quad \frac{dy}{dx}$ $\rightarrow 2x + y - 3\frac{dy}{dx} + x\frac{dy}{dx} = 0$

$x\frac{dy}{dx} - 3\frac{dy}{dx} = -2x - y \rightarrow \frac{dy}{dx}(x-3) = -2x - y \rightarrow \frac{dy}{dx} = \frac{-2x-y}{x-3}$

$(2,1) \rightarrow \frac{-2(2)-1}{2-3} \rightarrow \frac{-4-1}{-1} \rightarrow \frac{-5}{-1} \rightarrow \boxed{5}$

(12) $w'(t) = -2t \rightarrow -2(3) = -6$

(14) $y = Ce^{kt}$

(16) $f'(x) = 12x - 6x^2$ $\quad$ 120 $\quad f''(x) = 12 - 12x$ $\qquad$ $12 - 6$

46 $\cup$

$6 - 1.5$ $\quad \frac{3}{2}$ $\quad 12 - 18$

$\rightarrow \nearrow$ $\quad \cap$

$-12e^{-t}(\sin t) + 12e^{-t}(\cos t) = 0$ | $\cos t - \sin t = 0$

$12e^{-t}(-\sin t + \cos t) = 0$

$\cancel{12e^{-t} = 0}$ $\quad (\sin + \cos t) = 0$

(18) $(\tan 5x)\sec(5x) - 1 = F(x)$

$-2\sin x \quad \frac{\pi}{2} \to -2\sin\frac{\pi}{2} \to -2$

$2\cos\frac{\pi}{2} + 1 \to \left(\frac{\pi}{2}, 1\right)$

$y - 1 = -2\left(x - \frac{\pi}{2}\right) \to y_? = -2\left(x - \frac{\pi}{2}\right) + 1$

$y_?(1.5) = -2\left(1.5 - \frac{\pi}{2}\right) + 1 \to -3 + \pi + 1$

$\to \pi - 2$

$\lim_{x \to 3} \frac{\ln x - \cancel{\ln 3}}{x - 3} \to \frac{\frac{1}{x}}{1} \to \frac{1}{3}$

$\frac{1}{2y} dy = \frac{1}{2x+1} dx \to \frac{1}{2}\ln\frac{1}{2x+1}$

$u' = 2$

$g(x) = -2x \qquad f(x) = \frac{1}{2}x$

$-2x \qquad \to \frac{1}{2}$

$-2x\left(\frac{1}{2}\right) + (2)\left(\frac{1}{2}x\right) \to \quad -x - x$

$\to -2x \to \quad -2(2) \to 4$

$g(x) = -2x + 5 \qquad f(x) = \frac{1}{2}x + 2$

$f'(x) = \frac{1}{2} \qquad g'(x) = -2$

$(-2x+5)\left(\frac{1}{2}\right) + \left(\frac{1}{2}x + 2\right)(-2) \quad @\, 2 \to (-4+5)\frac{1}{2} + (3)(-2)$

$\to \frac{1}{2} + -6 \to -\frac{11}{2}$

$A=\frac{1}{2}s^2$ $s=\sqrt{32}$ $h=\sqrt{64}$

$\frac{dA}{dx} = +12\ cm^2/s$ find $\frac{ds}{dx}$ $A=\frac{1}{2}32=16$

$\frac{12}{8}$ 6

$\frac{ds}{dx} = $ → $\frac{ds}{dx} = 12\ cm/s$

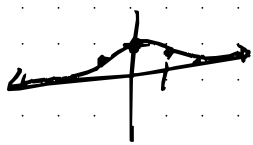
yuck

$12^2+12^2=c^2$

$144+144=c^2$

$288=\sqrt{288}$

$\frac{1}{x^2+1}$ $\frac{1}{x^3+1}$ $\frac{1}{e^x-1}$ $\frac{1}{e^x+1}$

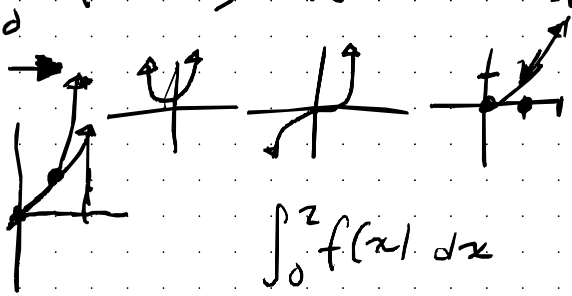


$x^{3\,\frac{1}{2}} \to x^{3/2}$

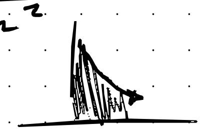
$\int_0^2 (x^3+1)^{1/2}\, x^2\, dx \to \int_0^2 \sqrt{x^3+1}\, x^2\, dx$

$2\sqrt{2} \to 2(1.?)$

$\to 2.?$



$\int_0^2 f(x)\,dx$ $\frac{2(n+1)(3n+2)}{n^2}$



$2(?) \to \frac{4}{0} \to \infty$

$4(5) \to \frac{20}{} \to 20$

$\frac{6(8)}{4} \to \frac{48}{4} \to 12 \to 12$

$\int 8x\sqrt{40-2x^2}\,dx$ $\quad u = 40-2x^2$

$\int 8\cancel{x}\sqrt{u}\,\frac{du}{-4\cancel{x}}$ $\quad du = -4x\,dx$ $\quad dx = \frac{du}{-4x}$

$-2\int \sqrt{u}\,du \to u^{1/2} \to -2\left[\frac{u^{3/2}}{3/2}\right]+C$

$\to \frac{-4\sqrt{40-2x^2}^{\,3}}{3}+C$

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \text{average value}$$

$$\frac{f(b)-f(a)}{b-a} \quad [a,b]$$

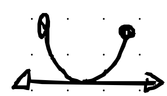
of $f(x) = x^2+4x-5 \quad [1,3]$

$^a f(1) = 1^2+4(1)-5 \to 0$

$^b f(3) = 3^2+4(3)-5 \to 9+12-5 \to 16$

$\frac{16-0}{3-1} \to \boxed{8}$

$f(x) = x^3-4 \quad [2,5]$



$f(2) = 2^3-4 \to 8-4 \to 4$

$f(5) = 125-4 \to 121$

$\frac{121-4}{5-2} \to \frac{117}{3} = 39$

$\int \frac{x^3}{(2+x^4)^2}\,dx$ $\quad u = 2+x^4$ $\quad du = 4x^3\,dx$ $\quad dx = \frac{du}{4x^3}$

$u^{-2} \to \frac{u^{-1}}{-1} \to -\frac{1}{u}$

$\int \frac{\cancel{x^3}}{(u)^2}\frac{du}{4\cancel{x^3}} \to \frac{1}{4}\int \frac{1}{u^2}\,du \to -\frac{1}{4}\left(\frac{1}{2+x^4}\right)+C$

Review of Mock Exam: MCQ 1

Memorize trapezoidal rule

$$\int_a^b f(x)\,dx = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + \dots 2f(x_{n-1})\right]$$

[Practice implicit differentiation

Find tangent at certain x-val on function

Integrating with u-sub

$$\frac{f(x) = x^3}{f'(x) = 3x^2}$$

$g'(2)$

g	$g(x)$
2	4

$$\frac{1}{3(2)^2} = \frac{1}{12}$$